R-Matrix Theory: Photon Channels

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Photon Channels

- Photon channels may also be described using R-matrix methods.
- Photon channels are labeled using p and
  – λ_f: the final state J^π, E_x,…
  – transition type:
  $\rightarrow$ E for electric ($\epsilon = 1$)
  $\rightarrow$ M for magnetic ($\epsilon = 0$)
  – multipolarity: $L = 1, 2, 3\ldots$
- The usual selection rules for electromagnetic transitions apply.
- The details of the implementation in R-matrix differ widely.

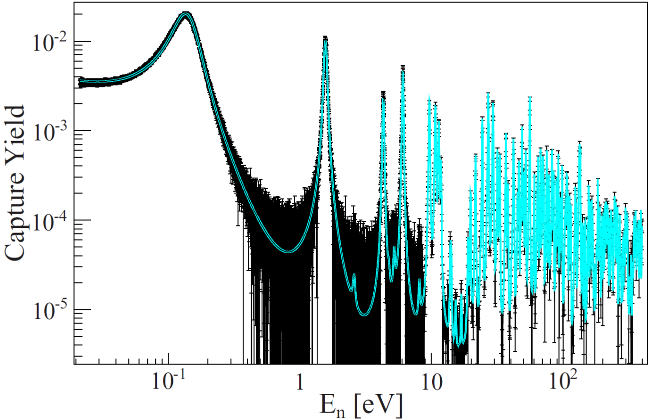
Heavy Nuclei

- For neutron capture on heavy nuclei, one is often presented with vary narrow resonances which are closely spaced.
- The photon channels may contribute significantly to the widths of levels $\rightarrow$ first-order perturbation theory is not applicable.
- The number final states may be very large, which requires some type of approximation. Most experiments do not resolve individual levels in the final state.

Heavy Nuclei, continued

- Reich-Moore Approximation: C.W. Reich and M.S. Moore, Phys. Rev. **111**, 929 (1958), `https://doi.org/10.1103/PhysRev.111.929`:
  A scheme to include channels where you do not specify the branching ratio among final states in those channels. Useful for treating photon and fission channels in heavy nuclei.
- This is implemented in the code R-matrix code `SAMMY`.

Neutron Capture in Heavy Nuclei



^{176}Lu(n,γ) measured with the DANCE detector at the Los Alamos Neutron Science Center. The blue curve is a SAMMY fit. O. Roig *et al.*, Phys. Rev. **93**, 034602 (2016), https://doi.org/10.1103/PhysRevC.93.034602. ^{176}Lu is an s-process branching point.

Light Nuclei

- The final states are usually resolved.
- First-order perturbation theory is applicable (Γ_γ is small compared to other partial widths). In fact, this may be violated far below the Coulomb barrier, but then a single Breit-Wigner resonance is an excellent approximation.
- The matrix element calculated in perturbation theory may depend on the long-ranged parts of the wavefunctions. For electric (EL) transitions, one has $T \propto \langle \Psi_i | r_\alpha^L | \Psi_f \rangle$, at least for the large-radius contribution.
- Using R-matrix wavefunctions, the matrix element is naturally split into *internal* and *external* contributions.

External Contribution: History

- This was first introduced by R.G. Thomas in 1952.
- Unambiguous experimental evidence for external capture was provided by the ^{3}He(α, γ) and ^{3}H(α, γ) cross sections measurements of H.D. Holmgren and R.L. Johnston in 1959.
- Further development, with a focus on neutron-induced reactions, was provided by A.M. Lane and J.E. Lynn in the early 1960s.
- An important milestone was the ^{17}O$(\gamma, n_0)^{16}$O calculations of R.J. Holt et al., Phys. Rev. C **18**, 1962 (1978), https://doi.org/10.1103/PhysRevC.18.1962.
- Development culminated with F.C. Barker and T. Kajino, Aust. J. Phys. **44**, 369 (1991), https://doi.org/10.1071/PH910369.
- There was a parallel development of *Direct Capture* models, which we will not discuss.

Barker-Kajino Formalism

This is what is done in the `AZURE2` code. See also:

- AZURE: An R-matrix code for nuclear astrophysics, R. E. Azuma, E. Uberseder, E. C. Simpson, C. R. Brune, H. Costantini, R. J. de Boer, J. Görres, M. Heil, P. J. LeBlanc, C. Ugalde, and M. Wiescher, Phys. Rev. C 81, 045805, 17 pages (2010).
  https://doi.org/10.1103/PhysRevC.81.045805

- The $^{12}\mathrm{C}(\alpha,\gamma)^{16}\mathrm{O}$ reaction and its implications for stellar helium burning, R. James deBoer, R.E. Azuma, A. Best, C.R. Brune C.E. Fields, J. Görres, S. Jones, M. Pignatari, D. Sayre, K. Smith, F.X. Timmes, E. Uberseder, and M. Wiescher, Reviews of Modern Physics, **89**, 035007 (2017), Sec IV.D.
  https://doi.org/10.1103/RevModPhys.89.035007

- These are good references for the formulas.

Barker-Kajino Formalism, continued

- The transition matrix connecting nuclear and photon channels may be defined in 1st-order perturbation theory as

$$T^J_{c\to p} = \left[\frac{8\pi(L+1)}{\hbar v_\alpha L}\right]^{1/2} \frac{k_\gamma^{L+1/2}}{(2L+1)!!} \times \langle \alpha s l J || i^{L+1-\epsilon} \mathcal{M}^{\epsilon L} || \lambda_f \rangle^* . \tag{1}$$

- The angle-integrated radiative capture cross section is then given by

$$\sigma_{\alpha\to\lambda_f} = \frac{\pi}{k_\alpha^2} \sum_{JlsL\epsilon} g_J \left| T^J_{c\to p} \right|^2 . \tag{2}$$

- The differential cross section can be calculated using *Angular distribution theory for particle-capture-γ reactions*, R. G. Seyler and H. R. Weller Phys. Rev. C **20**, 453 (1979), https://doi.org/10.1103/PhysRevC.20.453.

Barker-Kajino Formalism, continued

- We proceed by splitting the transition matrix into internal and external parts:

$$T^J_{c\to p} = T^J_{c\to p}(\text{int}) + T^J_{c\to p}(\text{ext}).$$

- The internal contribution is

$$T^J_{c\to p}(\text{int}) = -2i\Omega_c (P_c k_\gamma^{2L+1})^{1/2} \sum_{\lambda\mu} A_{\lambda\mu}\gamma_{\mu c}\gamma_{\lambda p}$$

- where

$$\gamma_{\lambda p} = \left[\frac{4\pi(L+1)}{L}\right]^{1/2} \frac{\langle\lambda||i^{L+1-\epsilon}\mathcal{M}^{\epsilon L}||\lambda_f\rangle_{\text{int}}}{(2L+1)!!}.$$

Barker-Kajino Formalism, continued

- For EL transitions, the external contribution is calculated using the known two-body wavefunctions for the bound and scattering states. This involves Coulomb functions and the scattering matrix of the non-photon channels.

- This contribution is also proportional to the ANC of the bound final state!

- Note that $T^J_{c\to p}(\text{int})$ and $T^J_{c\to p}(\text{ext})$ have the same complex phase. This phase is determined by scattering matrix of the non-photon channels (Watson's Theorem): L. D. Knutson Phys. Rev. C **59**, 2152 (1999), https://doi.org/10.1103/PhysRevC.59.2152.

Barker-Kajino Formalism, continued

- For many non-resonant reactions, particularly if they have low Q value, the external contribution is large and dominates. This is the case for $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$, $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$, and $^{16}\mathrm{O}(p,\gamma)^{17}\mathrm{F}$.

- The analysis of radiative capture can also be used to determine ANCs. In some cases, a level below the threshold may be important as both a *subthreshold resonance* and as a final state for external capture.

- In a phenomenological analysis, the $\gamma_{\lambda p}$ are simply fit parameters.

- Note that the external contribution is similar to what one would calculate in a "Direct Capture Model" but it is not exactly the same.

Thank you for your attention.